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Numerical Methods - MA 207
Interpolation

1. Let the function $y = f(x)$ take the values $y_0, y_1, \dots, y_n$ corresponding to the values $x_0, x_0 + h, \dots, x_0 + nh$ of x. Suppose $f(x)$ is a polynomial of degree n and it is required to evaluate $f(x)$ for $x = x_0 + ph$, where p is a any real number. Derive Newton's forward difference interpolation formula, by using shift operator E. [Hint : $y_p = f(x) = f(x_0 + ph) = E^p f(x_0) = (1 + \Delta)^p y_0$.]

2. The table gives the distance in nautical miles of the visible horizon for the given heights (in feet) above the earth's surface.

x	100	150	200	250	300	350	400
$y = f(x)$	10.63	13.03	15.04	16.81	18.42	19.90	21.27

Find the values of y when $x = 160$ and $x = 410$.

3. From the following table, estimate the number of students who obtained marks between 40 and 45.

Marks	30-40	40-50	50-60	60-70	70-80
No. of Students	31	42	51	35	31

4. Find the cubic polynomial which takes the following values.

x	0	1	2	3
$y = f(x)$	1	2	1	10

Also compute $f(4)$.

5. In the table below, the values of y are consecutive terms of a series of which 23.6 is the 6th term. Find the first and tenth terms of the series.

x	3	4	5	6	7	8	9
$y = f(x)$	4.8	8.4	14.5	23.6	36.2	52.8	73.9

6. Using Newton's forward interpolation formula, show that

$$\sum_{k=1}^{n} k^3 = \left\{ \frac{n(n+1)}{2} \right\}^2.$$

7. Find the polynomial $f(x)$ by using Lagrange's formula and hence find $f(3)$ for

x	0	1	2	3
$y = f(x)$	2	3	12	147

8. A curve passes through the points $(0, 18), (1, 10), (3, -18)$ and $(6, 90)$. Find the slope of the curve at $x = 2$.

9. Using Lagrange's formula, express the function

$$\frac{3x^2 + x + 1}{(x-1)(x-2)(x-3)}$$

as a sum of partial fractions.

10. Find the missing term in the following table using interpolation.

x	0	1	2	3	4
$y = f(x)$	1	3	9	-	81

11. Find the distance moved by a particle and its acceleration at the end of 4 seconds, if the time verses velocity data is as follows.

t	0	1	3	4
v	21	15	12	10

12. Using Lagrange's formula prove that

$$y_0 = \frac{y_1 + y_{-1}}{2} - \frac{1}{8}\left\{\frac{1}{2}(y_3 - y_1) - \frac{1}{2}(y_{-1} - y_{-3})\right\}.$$

[Hint : Here $x_0 = -3, x_1 = -1, x_2 = 1, x_2 = 3$.]

13. Given

$$\begin{aligned} \log_{10} 654 &= 2.8156, \quad \log_{10} 658 = 2.8182, \\ \log_{10} 659 &= 2.8189, \quad \log_{10} 661 = 2.8202 \end{aligned}$$

find by using Lagrange's formula, the value of $\log_{10} 656$.

14. The following table gives the viscosity of an oil as a function of temperature. Use Lagrange's formula to find viscosity of oil at a temperature of 140°.

Temperature	110°	130°	160°	190°
Viscosity	10.8	8.1	5.5	4.8

15. Given $u_1 = 40, u_3 = 45, u_5 = 54$, find u_2 and u_4.

16. Given $y_0 = 3, y_1 = 12, y_2 = 81, y_3 = 200, y_4 = 100, y_5 = 8$, without forming the difference table, find $\Delta^5 y_0$.

17. From the data given below, find the number of students whose weight is between 60 and 70.

Weight	0-40	40-60	60-80	80-100	100-120
No. of Students	250	120	100	70	50

18. The values of $U(x)$ are known at a, b, c. Show that maximum or minimum of Lagrange's interpolation formula is attained at

$$x = \frac{\sum U_a(b^2 - c^2)}{2\sum U_a(b - c)}.$$

19. By iterative method, tabulate $y = x^3$ for $x = 2, 3, 4, 5$ and calculate the cube root of 10 correct to 3 decimal places.

20. The following values of $y = f(x)$ are given

x	10	15	20
y	1754	2648	3564

Find the value of x for $y = 3000$ by iterative method.

21. Using inverse interpolation, find the real root of the equation $x^3 + x - 3 = 0$ which is close to 1.2.

22. Solve the equation $x = 10 \log x$, by iterative method, given that

x	1.35	1.36	1.37	1.38
$\log x$	0.1303	0.1355	0.1367	0.1392

23. Apply Lagrange's method, to find the value of x when $f(x) = 15$ from the given data.

x	5	6	9	11
$f(x)$	12	13	14	16

24. The equation $x^3 - 15x + 4$ has a root close to 0.3, obtain this root upto 4 decimal places using inverse interpolation.

25. If $f(x) = \frac{1}{x^2}$, find the first divided differences

    (a) $[a, b]$ (b) $[a, b, c]$.

    Here a, b, c are arguments for $f(x) = \frac{1}{x^2}$.

26. Find the divided difference table for the function $f(x) = x^2 + 2x + 2$ whose arguments are $1, 2, 4, 7, 10$.

27. Find the following divided differences of $f(x) = \frac{1}{x^2}$ whose arguments are

    (a) $[1, 2]$ (b) $[1, 2, 4]$ (c) $[1, 2, 4, 5]$ (d) $[2, 4, 5]$.

28. If $f(x) = \frac{1}{x}$ whose arguments are a, b, c, d in this order, prove that

$$[a, b, c, d] = \frac{-1}{abcd}.$$

29. Using Newton's divided difference formula, find the equation of the cubic curve which passes through the points $(4, -43), (7, 83), (9, 327)$ and $(12, 1053)$. Hence find $f(10)$.

30. Given the values

x	5	7	11	13	17
$y = f(x)$	150	392	1452	2366	5202

    evaluate $f(9)$, using

    (a) Lagrange's formula
    (b) Newton's divided difference formula.

31. Find the value of x correct to one decimal place for which $y = 7$ given

x	1	3	4
$y = f(x)$	4	12	19

32. Tabulate $y = x^3$ for $x = 2, 3, 4, 5$ and calculate the cube root of 10 correct to 3 decimal places.

33. Using the Newton's divided difference formula, evaluate $f(8)$ and $f(15)$ given

x	4	5	7	10	11	13
$y = f(x)$	48	100	294	900	1210	2028

34. Determine $f(x)$ as a polynomial in x for the following data.

x	-4	-1	0	2	5
$y = f(x)$	1245	33	5	9	1335

35. Using Newton's divided difference formula, find the missing value from the following table.

x	1	2	4	5	6
$y = f(x)$	14	15	5	-	9

36. Interpolate $f(2)$ from the following data

x	1	2	3	4	5
$f(x)$	7	?	13	21	37

and explain why the values obtained is different from the obtained by putting $x = 2$ in the expression $2^x + 5$.

37. From the following table of yearly premiums for policies maturity at quinquennial (recurring every five years) ages, interpolate the premiums for policies maturity at the age of 12 years.

Age (years) x	10	15	20	25	30	35
Premimum $f(x)$	3.54	3.22	2.91	2.60	2.31	2.04

38. The population of a country is given below. Estimate the population for the year 1965.

Year (t)	1931	1941	1951	1961	1971
Population (U_t) (in thousands)	46	66	81	93	101

39. The following are the marks obtained by 492 candidates in a certain examination.

x	0-40	40-45	45-50	50-55	55-60	60-65
$f(x)$	210	43	54	74	32	79

Find out the number of candidates

(a) who secured more than 48 but not more than 50 marks

(b) less than 48 but not less than 45 marks.

[Hint: For the marks-range $a - b$, define x as $\frac{b-40}{5}$.]

40. If p, q, r, s are the successive entries corresponding to equidistant arguments in a table, show that when 3rd differences are taken into account, the entry corresponding to the argument half-way between the arguments of q and r is $A + \frac{1}{24}B$, where A is the arthimetic mean of q and r, and B is the arithmetic mean of $3q - 2p - s$ and $3r - 2s - p$.

41. The following are the mean temperatures (Fahreheit) on three days, 30 days apart round the periods of summer and winter. Estimate the appropriate dates and values of the maximum and minimum temperatures.

Day	Summer		Winter	
	Date	Temperature	Date	Temperature
0	15th June	58.8	16th Dec	40.7
30	15th July	63.4	15th Jan	38.1
60	14th Aug	62.4	14th Feb	39.3

[Hint : Form difference tables for summer and winter separately, by considering the transformation $d(\texttt{day}) \to d/30 = x$.]

42. Use Newton's forward difference formula to obtain the interpolating polynomial $f(x)$ satisfying the following data.

x	1	2	3	4
$f(x)$	26	18	4	1

If another point $x = 5, f(x) = 26$, is added to the above data, will the interpolating polynomial, be the same as before or different. Explain why?

43. Given $\sum_{x=1}^{10} f(x) = 500426$, $\sum_{x=4}^{10} f(x) = 329240$, $\sum_{x=7}^{10} f(x) = 175212$ and $f(10) = 40365$. Find $f(1)$. [Hint : Define $u_t = \sum_{x=t}^{10} f(x)$, for $t = 1, 4, 7, 10$. Then $f(1) = u_1 - u_2$. Find u_2.]

44. Given $f(0) = 1, f(1) + f(2) = 10$ and $f(3) + f(4) + f(5) = 65$, find $f(4)$. [Hint: Consider $f(x) = a + bx + cx^2$.]

45. Find $f[3, 4, 5, 6]$ when $f(x) = x^3 - x$.

46. Given that $f(0) = 8, f(1) = 68, f(5)123$. Construct a divided difference table. Using the table determine the value of $f(2)$.

47. Form the divided difference table and find $f[a, b, c]$ for $f(x) = 1/x$.

48. Find the polynomial of the lowest degree which assumes the values $3, 12, 15, -21$ when x has the values $3, 2, 1, -1$ respectively.

49. Given $f(0) = -18, f(1) = 0, f(3) = 0$, $f(5) = -248$, $f(6) = 0$ and $f(9) = 13104$, find the form of $f(x)$, assuming it to be a polynomial in x.

50. The values of $f(x)$ are known at points x_0, x_1, x_2. Prove that the second divided difference is equal to

$$\frac{f(x_0)}{(x_0 - x_1)(x_0 - x_2)} + \frac{f(x_1)}{(x_1 - x_0)(x_1 - x_2)} + \frac{f(x_2)}{(x_2 - x_0)(x_2 - x_1)}.$$

Write down the equal form of nth divided difference.

51. Show that Lagrange's formula can be evolved by equating $(n+1)$th divided differences of $f(x)$ to zero if $f(x)$ is a polynomial of degree n. [Hint: $0 = f[x, x_0, x_1, \cdots, x_n]$ and the previous result for nth divided difference]

52. Prove that Lagrange's formula can be put in the form

$$p_n(x) = \sum_{k=0}^{n} \frac{\pi(x) f(x_k)}{(x - x_k)\pi'(x_k)}$$

where

$$\pi(x) = \Pi_{r=0}^{n}(x - x_r) \text{ and } \pi'(x_k) = \left[\frac{d}{dx}\pi(x)\right]_{x=x_k}.$$

53. The following values of the function $f(x)$ for values of x are given $f(1) = 4, f(2) = 5, f(7) = 5$ and $f(8) = 4$. Find the values of $f(6)$ and also the value of x for which $f(x)$ is maximum or minimum.

54. Apply Newton's divided difference formula (inversely) to find, to two decimal places, the value of x when $y = f(x) = 19$, from the following data.

x	0	1	2
$f(x)$	0	1	20

55. The mode of a certain frequency curve $y = f(x)$ is very near to $x = 9$ and the values of the frequency density $f(x)$ for $x = 8.9, 9$ and 9.3 are respectively equal to $0, 30, 0.35$ and 0.25. Calculate the approximate value of mode.

56. The points $(7,3), (8,1), (9,1), (10,6)$ satisfy the function $y = f(x)$. Use Lagrange's interpolation formula, to find y for $x = 9.5$, and also find the interpolating polynomial.

57. Obtain the value of x for $y = 30$ by successive approximation method from the following data.

x	10	12	14	16
$f(x)$	25	32	40	50

58. Apply Lagrange's formula (inversely) to find the value of x when $y = 6$, given the following table.

x	168	120	72	63
$f(x)$	3	7	9	10

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